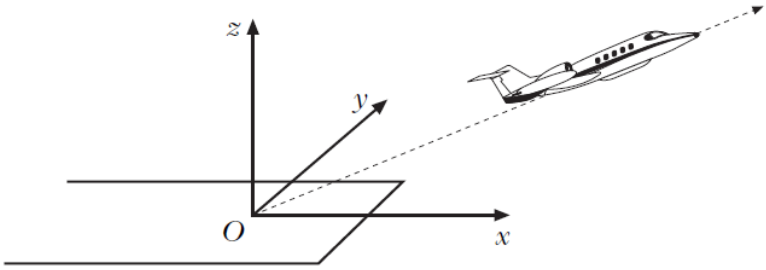
Closest Approach/Collisions

2007	**A8.** A car and a motorcycle are travelling along straight, horizontal roads which intersect at right angles at a point O. The car is travelling northwards at a constant speed, while the motorcycle is travelling eastwards at twice the speed of the car. At the instant when the motorcycle passes through O, the car is 40 metres south of O. Calculate the minimum distance between the car and the motorcycle.	**7**
2008	**A6.** At 12 noon, an aircraft is above a point A and is flying due West at a uniform speed of 180 km h^{-1}. Thirty minutes later, a second aircraft, which is flying at exactly the same height as the first with a uniform speed of 240 km h^{-1}, is 60 km due south of A. The aircraft are on a collision course. (a) Calculate the time when the collision would take place if no evasive action were taken. (b) Calculate the bearing on which the second aircraft is travelling.	**4** **2**
2011	**A4.** Relative to a rectangular coordinate system with origin O the position vector of a passenger aircraft is $-100\mathbf{i} + 250\mathbf{j}$, at 09.00 hours, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the Ox and Oy directions. The aircraft is travelling with uniform velocity $300\mathbf{i} + 400\mathbf{j}$. Relative to the same coordinate system, a military aircraft travelling with uniform velocity $600\mathbf{i} + 500\mathbf{j}$, has position vector $-100\mathbf{i} + 400\mathbf{j}$ at 09.30 hours. In these expressions, the distances are measured in kilometres and speeds in kilometres per hour. Show that the two aircraft are on a collision course.	**5**
2012	**A8.** A particle P is projected so that its position vector is given by $(t^2 + 3)\mathbf{i} + 4t\mathbf{j}$. The time is measured in seconds, distances are measured in metres and $\mathbf{i}$, $\mathbf{j}$ are the unit vectors in the directions of rectangular axes Ox and Oy respectively. A second particle Q has the same acceleration as the particle P and, at time $t = 0$, the particle Q has velocity $(-4\mathbf{i} + \mathbf{j})$ and position vector $8\mathbf{j}$. Find: (a) an expression, in terms of t, for the position vector of Q; (b) the time taken from the start of the motion until the particles are closest to each other; (c) the time at which the particles are moving at right angles to each other.	**3** **4** **3**
2014	**A6.** At 3pm, a patrol vessel travelling at a constant speed of 20 km h^{-1} sights a ship 15 km away to the North East. The ship is travelling due North at a constant speed of 10 km h^{-1}. Find the bearing on which the patrol vessel should travel to intercept the ship and the time at which this will occur.	**6**
2015	**A7.** At 3pm, a yacht is travelling with velocity $(4\mathbf{i} + 20\mathbf{j})$ km hr^{-1} while a trawler, positioned due north of the yacht, is travelling with a velocity $(-3\mathbf{i} - 4\mathbf{j})$ km hr^{-1}. If the closest they get to each other in the subsequent motion is 4·2 kilometres, find the time to the nearest minute when they are closest and how far apart they were originally.	**6**

2010

A10. Relative to the rectangular coordinate system as shown in the diagram, a horizontal runway is aligned along the Ox direction. The unit vectors, **i**, **j** and **k** are in the x, y and z directions respectively.



Aircraft A takes off from a point O on the runway and thereafter climbs with constant speed $V\,\mathrm{m\,s^{-1}}$ at an angle of $45°$ to the horizontal in the x-z plane.

(a) Find the position vector of the aircraft A, t seconds after it takes off. **2**

A second aircraft B is travelling with a constant velocity vector

$$\mathbf{v}_B = \frac{V}{\sqrt{2}}\mathbf{j}.$$

At the moment that aircraft A takes off, the position vector of B is

$$\mathbf{r}_B = L(\mathbf{i} - \mathbf{j} + 4\mathbf{k}),$$

where L is a positive constant.

(b) Find the position vector of aircraft B, t seconds after aircraft A has taken off. **2**

(c) Show that the distance D metres, between the aircraft is given by

$$D^2 = \tfrac{3}{2}V^2t^2 - 6\sqrt{2}VLt + 18L^2.$$ **3**

Hence, find the minimum distance between the aircraft, in terms of the constant L. **3**

2016 SP

14. Three vessels A, B and C are being tracked by coastguards at half-hour intervals. With distances measured in kilometres and speeds in kilometres per hour, they have the following displacement and velocity vectors:

Vessel	A	B	C
Time	10:00	10:30	11:00
Position	$2\mathbf{i}+7\mathbf{j}$	$6\mathbf{i}+9\mathbf{j}$	$12\mathbf{i}+9\mathbf{j}$
Velocity	$4\mathbf{i}+5\mathbf{j}$	$3\mathbf{i}+4\mathbf{j}$	$2\mathbf{i}+6\mathbf{j}$

(a) Show that if A and C continue without changing course they will collide.

Find the time and position of the collision. **5**

At the instant of the collision, vessel B changes course and then proceeds directly to the scene of the collision at its original speed.

(b) Find the time, to the nearest minute, at which vessel B will arrive at the scene of the collision and state the bearing of its course to this point. **5**